CURRICULUM, PEDAGOGY AND BEYOND

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The Role of Metacognition in Mathematical Problem Solving

Emiritus Professor Dianne Siemon
& Dr Kathy Arnold

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Problem Solving in the Curriculum

Students engage in mathematical problem solving when:

- they are presented with a problem situation for which they do not immediately know the answer,
- work through a process of planning, choosing and applying strategies, reviewing and analysing solutions,
- formulate situations mathematically,
- draw on previously learnt concepts, skills, procedures, and processes,
- evaluate, interpret and communicate solutions, and
- justify the reasonableness of their approaches in terms of the situation.

(ACARA, 2022; VCAA, 2024)

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Problem Solving?

1)	6.928 − 2.365	2)	78.07 − 43.55	3)	91.24 − 85.76	4)	670.2 − 158.8
5)	5.037 − 2.475	6)	71.25 − 67.89	7)	30.37 − 9.75	8)	67.2 − 38.45
9)	4.172 − 0.684	10)	62.90 − 37.67	11)	8.730 − 2.266	12)	651.6 − 281.3
13)	8.403 − 1.675	14)	572.1 − 485.3	15)	79.83 − 54.61	16)	972.8 − 565.4
17)	7.021 − 4.968	18)	8.38 − 3.725	19)	40.08 − 28.76	20)	6.731 − 3.482
21)	4.506 − 2.758	22)	92.7 − 16.49	23)	80.02 − 36.8	24)	7.206 − 4.564

Computer Programs/Apps

Work Sheets

Name : _____ Score : _____

Teacher : _____ Date : _____

What is the Fraction of the Shaded Area ?

1) _____ 6) _____

2) _____ 7) _____

3) _____ 8) _____

4) _____ 9) _____

5) _____ 10) _____

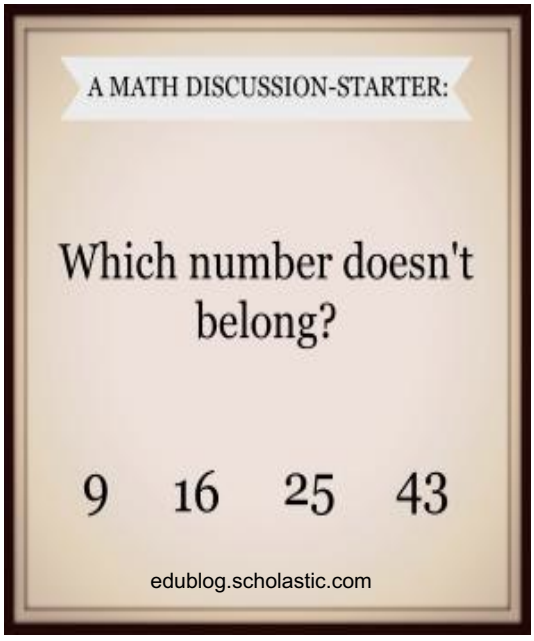
Problem Solving?

Note your first response.

Share with a neighbour.

What do you notice?

Crucial to any problem-solving experience is the task itself. (Kulm, 1979, p. 1)

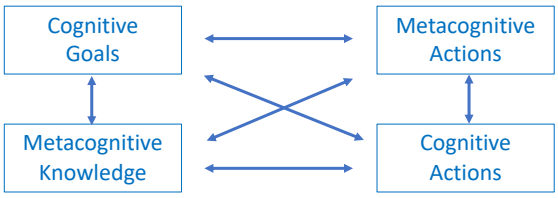


Metacognition

Metacognition – "one's knowledge of one's own cognitive processes and products or anything related to them … [including] the active monitoring and consequent orchestration of those processes" (Flavell, 1976, p. 243)

Metacognition is an awareness and understanding of one's own thought processes … often referred to as 'thinking about thinking'

Metacognition – cognitive awareness and self-regulation



Flavell's Model of Cognitive Monitoring (Flavell, 1981)

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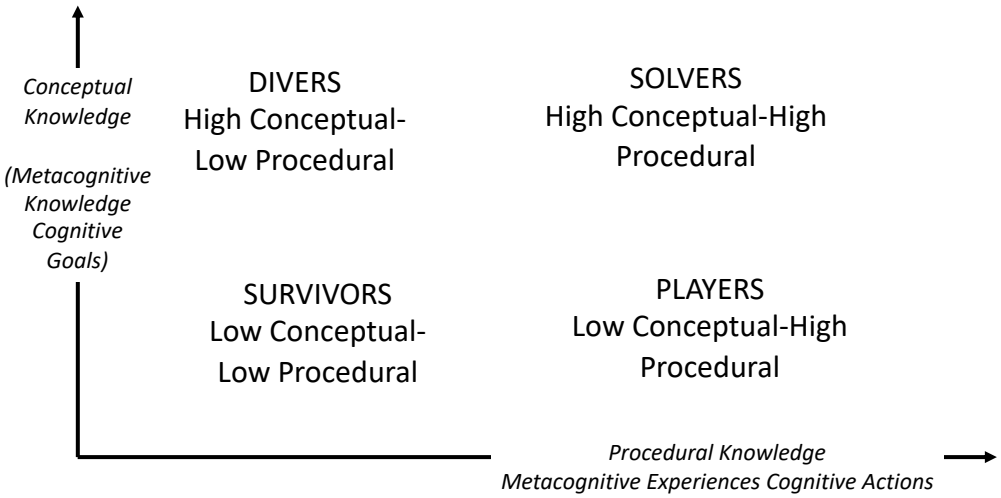
The role of metacognition



Meta-Model of Children's Approaches to Mathematical Problem Solving (Siemon, 1993)

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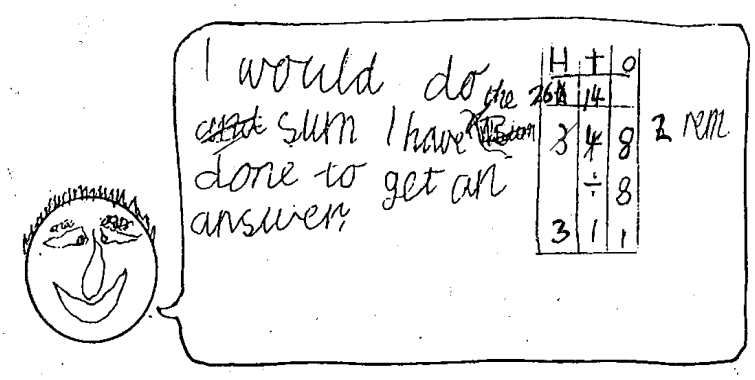
A 'Player'

Eight families shared a Tattslotto win of $348. How much did each family receive? (Year 4)



Nick's 'Talking Head' response (Siemon, 1993)

A 'Survivor'

Richard's solution strategy for 19 + 27

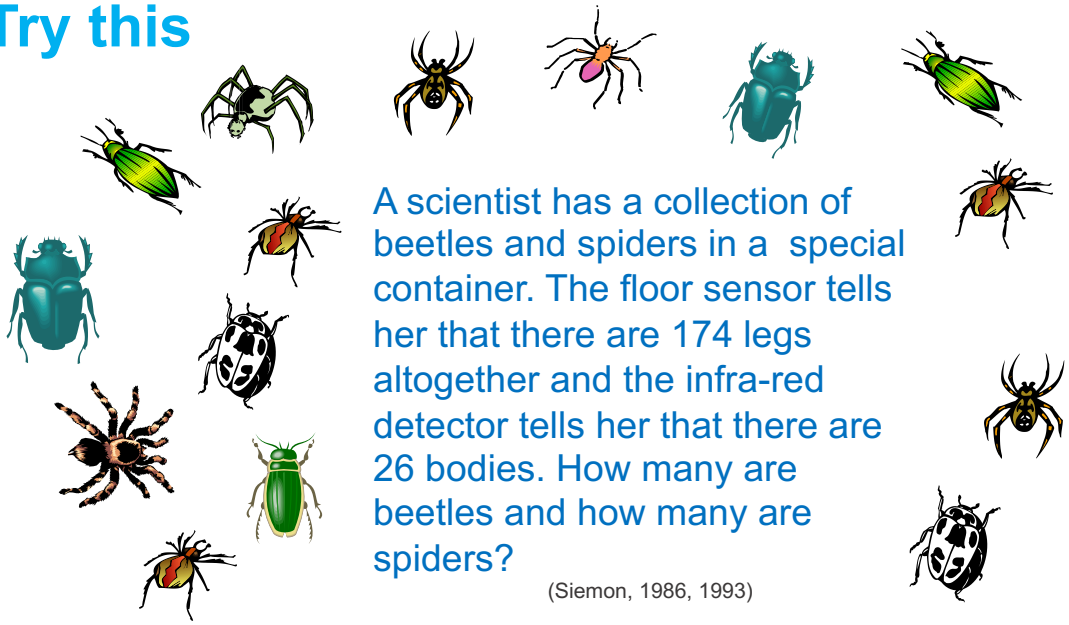
```
  19
+ 27
  31
  43
 424

  46
```

Can you identify Richard's strategy?

What does this response say about what Richard (Year 6) knows and values?

Source: Year 6 Classroom teacher (Siemon, 1993)

Try this



A scientist has a collection of beetles and spiders in a special container. The floor sensor tells her that there are 174 legs altogether and the infra-red detector tells her that there are 26 bodies. How many are beetles and how many are spiders?

(Siemon, 1986, 1993)

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Problem

No immediately available solution strategy

PROBLEM TYPE

Process

Re read the problem, record the data, what is required, UNDERSTAND

THINK about the problem, have I seen something like this before

TRY something

CHECK to see if it worked or not, does it satisfy the problem conditions

Strategies

Trial and error

Make a model

Draw a diagram

Use equations

If … then reasoning

Table of values

(Siemon, 1984, 1993)

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Problem Solving

The totality of processes engaged in by a group or individual as they attempt to negotiate a satisfactory resolution to a situation, task or question for which they had no immediately available, valid solution strategy or procedure at the outset.

(Siemon, 1984, 1993)

Implications for teaching – need to consider:

- Situations, tasks … PROBLEM TYPE
- Problem solving as a PROCESS
- Possible solution STRATEGIES

Problem Type

Recognising problem type and/or structure can help identify an appropriate solution strategy.

Problems can vary according to:

- the nature (transparency) of the information provided,
- the extent (amount) of information provided,
- the number of steps involved, and
- the underlying structure.

Part	Part
Whole	

Additive Structure

12 strawberry plants per row, 18 rows, how many plants?

M_1	M_2
1	12
18	?

Multiplicative Structure

Process

Polya's (1945) Model for effective Problem Solving

1. *Understanding the Problem*
2. *Devising a Plan*
3. *Carrying out the Plan*
4. *Looking Back*

Use **one ball** of play dough to represent 7 quarters.

You must use the entire ball.

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The ASK-THINK-DO Problem Solving Cycle

Questions we can ASK
What is the problems asking?
What do we need to find?
What information is given?
What do we need to know?
Does it make sense?
How could we check?
Have we forgotten anything?
Can we do it another way?

Things we can THINK about
Have I seen anything like this before?
How can I represent the problem?
Is it a part, or is it a whole?
What do I know?
Is there a similar problem we could use?
How can I organise the information?
What happened first?
What needs to be done next? Does it make sense?
What is not needed?
Where can I start?

Things we can DO
Make a list. Record data.
Draw a diagram.
Check information
Re-read the problem
Make a model
Use a simpler or similar problem.
Classify, Order, Compare.
Estimate. Work backwards.
Eliminate options.
Add, subtract, multiply, divide

(Siemon, 1984, 1993)

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Strategies

- Draw a picture.
- Restate the problem.
- Classify or order data.
- Act it out.
- Ask. "Is it a part or is it a whole?"
- Record data
- Estimate
- Use a pattern
- Make a model
- Experiment
- Use a simpler problem
- Make a table
- Draw a graph
- Make a list
- Eliminate options
- Assume a solution and work backwards
- Use symbols
- Write an equation

Butterflies
2 drops of nectar are needed to feed 5 butterflies. How many butterflies could be fed with 12 drops of nectar? How many drops of nectar are needed to feed 63 butterflies?

(Siemon, 1984, 1993)

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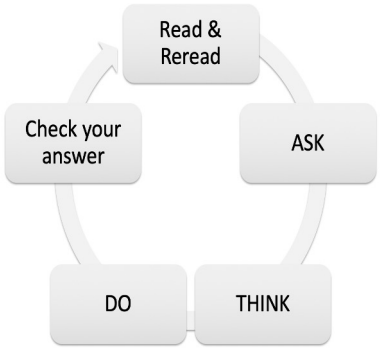
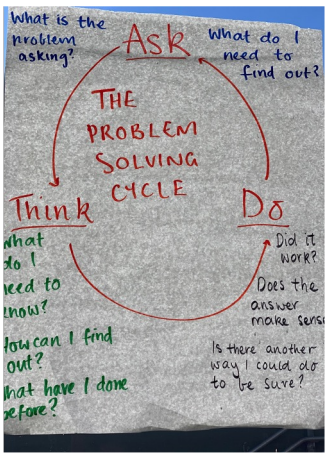
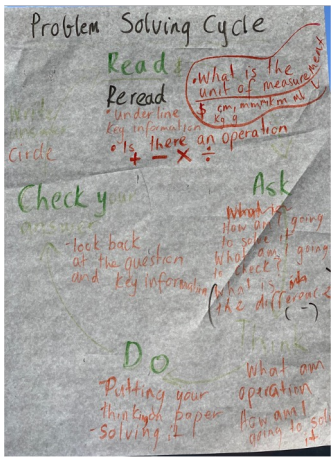
ASK-THINK-DO in practice
Not a Recipe!







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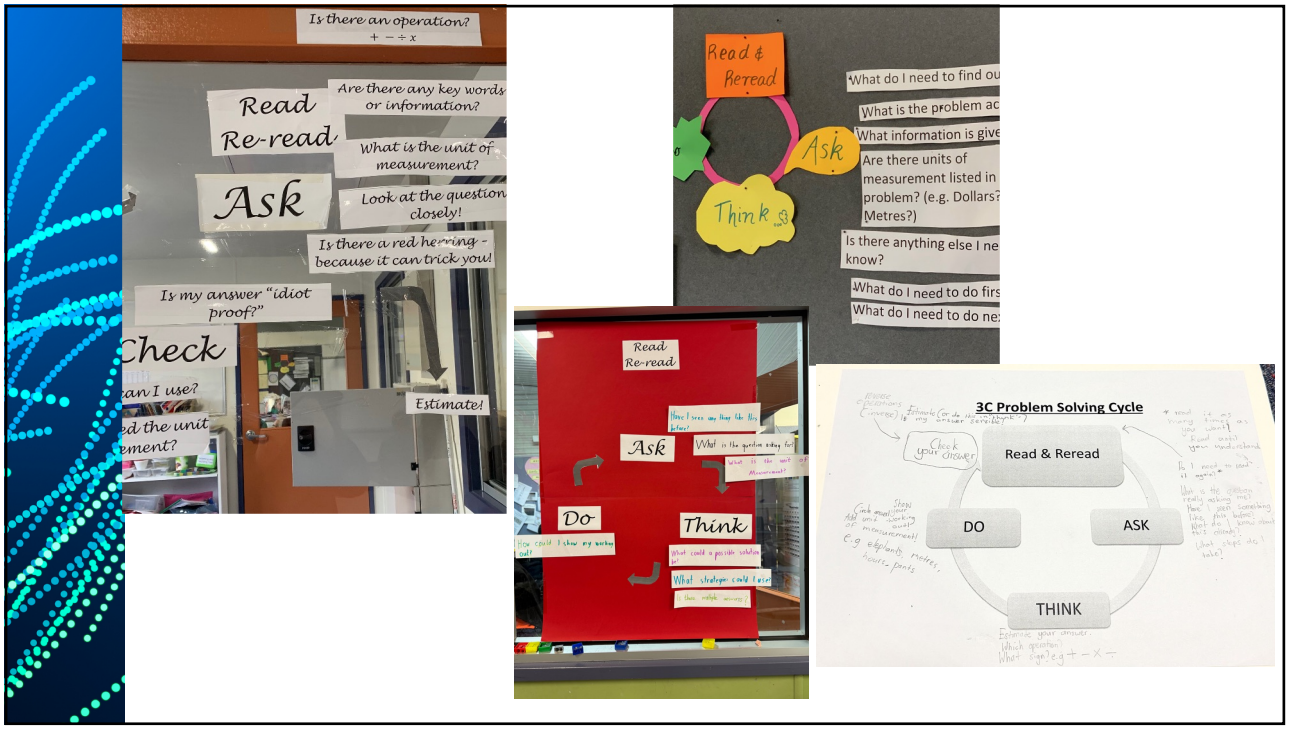

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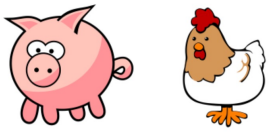
Choose/develop problems responsively

A scientist has a collection of beetles and spiders in a special container. The floor sensor tells her that there are 174 legs altogether and the infra-red detector tells her that there are 26 bodies. How many are beetles and how many are spiders?

A mad scientist has a collection of beetles and spiders in a special container. The floor sensor tells her that there are 68 legs altogether and the infra-red detector tells her that there are 10 bodies. How many beetles and how many spiders are there?

Farmer Fiona has pigs and chickens. Last Tuesday, she counted 34 eyes and 46 feet. How many chickens does she have?



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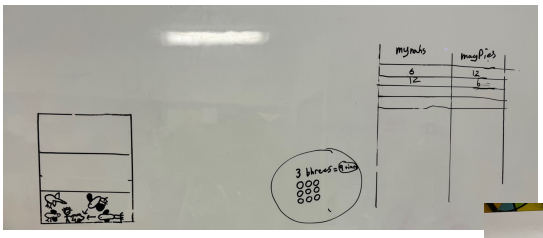
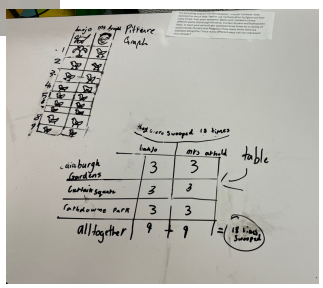
Allow the teaching point to evolve!





It is swooping season! On the weekend, I couldn't believe I was swooped so much that I had to use multiplication to figure out how many times I had been swooped. Banjo and I walked in three different parks (Edinburgh Gardens, Curtain Square and Rathdowne Park). In each park we both got swooped three times by a variety of birds (mostly Mynahs and Magpies.) How many times were we swooped altogether? How many different ways can you represent your answer?

Fluff!

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Problem solving can and should be a part of every mathematics lesson

An activity/task that does not result in student's thinking is not worth doing

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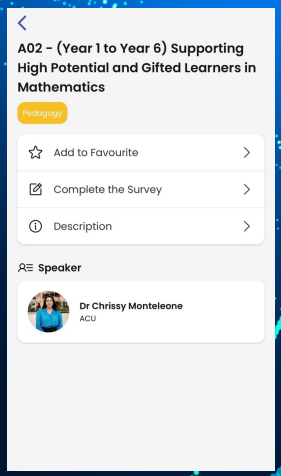
Be in it to WIN!



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